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APPLICATION OF THE METHOD OF INTEGRATING MATRICES FOR CALCULATING THE NATURAL OSCILLATIONS OF A PROPELLER BLADE WITH CONSIDERATION OF DEFLECTION IN TWO PLANES AND TWISTING

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APPLICATION OF THE METHOD OF INTEGRATING MATRICES FOR CALCULATING THE NATURAL OSCILLATIONS OF A PROPELLER BLADE WITH CONSIDERATION OF DEFLECTION IN TWO PLANES AND TWISTING

A.Yu. Liss and G.U. Margulis

The problem of the natural oscillations of a propeller blade has been solved by a number of authors (cf., for example, [1] - [4]). However, they have considered only displacement in the sweeping plane and twisting (separately or together). In contrast to these works, an account of the technique for calculating the natural oscillations of a blade with consideration of deflection in two planes and twisting is given here. /30*

The equations for such blade oscillations were obtained in [5]:

$$A_q(\bar{x}'', \bar{y}'', \bar{\theta}') - p^2 B_q(\bar{x}'', \bar{y}'', \bar{\theta}') = 0 \quad (q = 1, 2, 3), \tag{1}$$

where $\bar{x}(r), \bar{y}(r), \bar{\theta}(r)$ are the amplitudes of the deflections of the blade axis along the x and y axes and twist of its cross section; r is the coordinate of the blade cross section measured along its axis; A_q, B_q are some integral-differential expressions linearly dependent on $\bar{x}''$, $\bar{y}''$, $\bar{\theta}'$ and having the dimensions of torque; the primes indicate differentiation with respect to r.

Investigation of the system (1) has shown that if the functions A_q and B_q are considered as components of the vector operators A and B and a special definition for the scalar product is introduced, then these operators will be linear and symmetric, the operator B is always positive, and the eigenfunctions of the system (1)

* Numbers in margin indicate pagination in original foreign text.

are orthogonal with respect to the energy of the operators A and B, while the form of the orthogonality conditions depends on the boundary conditions of the problem.

For numerical solution of the problem for the eigenvalues described by the system (1), we use the widely applied method of replacing the system of functional equations (1) by a system of algebraic equations. For this purpose we write for selected blade cross sections r_i $(i=0, 1, \ldots, n)$ the equations (1) and boundary conditions, expressing A_q and B_q through the quantities $\bar{x}_i''$, $\bar{y}_i''$, $\bar{\theta}_i'$ using the formulae of numerical integration.

As a result we obtain a homogeneous system of algebraic equations with the parameter p^2 relative to the unknowns $\bar{x}_i''$, $\bar{y}_i''$, $\bar{\theta}_i'$.

This system of equations has the appearance in matrix form:

$$MZ - p^2 NZ = 0, \qquad (2)$$

where M and N are square matrices of the equation coefficients; /31

$$Z = \{X''Y''\Theta'\} \qquad (3)$$

is the column matrix of the unknowns.

By multiplying Equation (2) from the left by the inverse of M, we obtain the most general form for the matrix eigenvalue problem:

$$(U - \lambda E)Z = 0, \qquad (4)$$

where

$$U = M^{-1}N, \qquad (5)$$

$$\lambda = 1/p^2, \qquad (6)$$

E is the diagonal unit matrix.

Methods for solving the eigenvalue problem for the homogeneous systems of algebraic Equations (2) or (4) have been well elaborated.* Programs accomplishing this procedure are also well-known, the most convenient of which for solving the considered problem is that compiled by V. G. Bun'kov. It provides for finding any number of eigenvalues and corresponding eigenvectors for the problem (4) by using iterations and the method of exhaustion.

The calculation of the elements of the matrices M and N constitutes the main difficulty. It can be most effectively accomplished using the integrating matrix method of M. B. Vakhitor [8].

We recall that, according to this method, with the transition from the functional to the matrix equations, the numerical integration corresponds to multiplying from the left by the integrating matrix, whose form depends on the form of the considered integral. Thus, for the integral:

$$A_i = \int_{r_i}^{r_0} y(\zeta)\, d\zeta,^{**} \tag{7}$$

the matrix form has the appearance

$$A = J_1 Y, \tag{7'}$$

where $A = \{A_0 A_1 \ \ldots \ A_n\}$ is the column matrix (vector) of the quantities A_i; $Y = \{y_0 y_1 \ \ldots \ y_n\}$ is the column matrix of the quantities y_i; J_1 is the integrating matrix of first order.

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*Cf., for example, [6], [7], where there is an extensive bibliography.

**The points are numbered from the end of the integration interval (r_0) toward its beginning (r_n).

By analogy, the integrals:

$$A_i = \int_{r_n}^{r_i} y(\zeta)\, d\zeta, \quad A_i = \int_{r_i}^{r_0} y(\zeta)(\zeta - r_i)\, d\zeta, \quad A_i = \int_{r_i}^{r_0} f(\zeta)[y(\zeta) - y(r_i)]\, d\zeta$$

correspond to the matrix formulae:

$$A = J_2 Y, \quad A = J_3 Y, \quad A = J_5(f)\, Y,$$

and the integral

$$A_i = \int_{r_i}^{r_0} f(\zeta)\, y(\zeta)\, d\zeta$$

corresponds to the matrix relation: /32

$$A = J_1 [f]\, Y,$$

where [f] is the diagonal matrix, whose elements are the quantities $f(r_0)$, $f(r_1)$, ..., $f(r_n)$.

The double integral corresponds to double multiplication by the integrating matrix. Thus, the integral:

$$A_i = \int_{r_i}^{r_0} d\zeta_1 \int_{r_n}^{\zeta_1} y(\zeta)\, d\zeta.$$

corresponds to:

$$A = J_1 J_2 Y.$$

Methods which are suitable for programming, for calculating the elements of the integrating matrices, both for continuous as well as for discontinuous integrand functions, and also for the case when the spacing at different portions of the integration interval is not the same, are given in [8].

For the application of the integration matrix method to the considered problem, we write the expressions for the blade deformations. The following equalities are valid with consideration of the boundary conditions {cf. [5], Formula (18)}:

$$\left.\begin{aligned} &\bar{x}'(r)=\bar{x}'(r_v)+\int_{r_v}^{r}\bar{x}''(r)\,dr,\ \bar{x}=\int_{r_v}^{r}\bar{x}'(r)\,dr,\\ &\bar{y}'(r)=\bar{y}'(r_h)+\int_{r_h}^{r}\bar{y}''(r)\,dr,\ \bar{y}=\int_{r_h}^{r}\bar{y}'(r)\,dr,\\ &\bar{\theta}(r)=\bar{\theta}(r_{ax})\ \int_{r_{ax}}^{r}\bar{\theta}'(r)\,dr, \end{aligned}\right| \tag{8}$$

where r_v, r_h, r_{ax} are the coordinates of the vertical, horizontal, and axial joints.

We first consider the simplest case of a blade rigidly fastened to the hub. In this case:

$$\bar{x}'(r_v)=\bar{y}'(r_h)=\bar{\theta}(r_{ax})=\bar{x}(r_v)=\bar{y}(r_h)=0$$

and, in place of the double-term formulae (8), we obtain single-term. They correspond to the matrix relations:

$$\left.\begin{aligned} &X'=J_2X'',\ X=J_2X',\\ &Y'=J_2Y'',\ Y=J_2Y',\ \Theta=J_2\Theta', \end{aligned}\right| \tag{9}$$

where the vectors:

$$X''\{\bar{x}_0''\ldots\bar{x}_n''\},\ Y''=\{\bar{y}_0''\ldots\bar{y}_n''\},\ \Theta'=\{\bar{\theta}_0'\ldots\bar{\theta}_n'\}\ldots \Big| \tag{9'}$$

For a jointed blade without swinging compensation $(k_1 = 0)$:

$$x''(r_v)=y''(r_h)=\theta(r_{ax})=0. \tag{10}$$

One can also obtain single-term formulae for it, if one introduces the vectors:

$$X''_j = \{\bar{x}''_0 \dots \bar{x}''_{n-1} \bar{x}'_n\}, \quad Y''_j = \{\bar{y}''_0 \dots \bar{y}''_{n-1} \bar{y}'_n\}, \tag{11}$$

and a modified integrating matrix of second order: /33

$$J_{2j} = \begin{bmatrix} j_{00} \dots j_{0\,(n-1)} & 1 \\ \cdots & \cdot \\ \cdots & \cdot \\ j_{n0} \dots j_{n\,(n-1)} & 1 \end{bmatrix}; \tag{11'}$$

which differs from J_2 in that the last column is replaced by ones.

Then the double-term formulae [8] will correspond to the single-term matrix formulae:

$$X' = J_{2j}\, X''_j, \quad X = J_2 X',$$
$$Y' = J_{2j}\, Y''_j, \; Y - J_2\, Y', \; \Theta = J_2\, \Theta'. \tag{12}$$

Finally, at the butt of the jointed blade with swinging compensation k_1, the following boundary conditions are valid {cf. [5], Formula (18)}:

$$\bar{x}''\,(r_v) = 0,$$
$$\bar{y}''\,(r_h) + P\;\bar{\theta}\,(r_{ax}) = 0, \tag{13}$$
$$k_1\,\bar{y}'\,(r_h) + \bar{\theta}\,(r_{ax}) = 0,$$

where

$$\bar{P} = -k_1 [(GI_c + NI_p \alpha_c / F)/EI_1]_{butt.} \tag{13'}$$

To obtain single-term matrix formulae in this case, which correspond to (8), we introduce the vectors:

$$X''_c = \{\bar{x}''_1 \dots \bar{x}''_n \bar{x}'_n\}, \quad Y''_c = \{\bar{y}''_1 \dots \bar{y}''_n \bar{y}'_n\}, \quad \Theta'_c = \{\bar{\theta}'_1 \dots \bar{\theta}'_n \bar{\theta}_n\}, \tag{14}$$

in which the elements $\bar{x}_0''$, $\bar{y}_0''$, $\bar{\theta}_0'$ equaling zero are omitted {cf. [5], Expression (16)}, and the modified integrating matrix:

$$J_{2c} = \begin{bmatrix} j_{01} \cdots j_{0n} & 1 \\ j_{11} \cdots j_{1n} & 1 \\ \vdots & \vdots \\ j_{n1} \cdots j_{nn} & 1 \end{bmatrix} \tag{15}$$

of second order, obtained from the matrix J_2 by eliminating the first column and introducing the last column consisting of ones.

Then, Formulae (8) will correspond to the matrix equalities:

$$\begin{aligned} X' &= J_{2c}X_c'', \quad Y' = J_{2c}Y_c'', \quad \Theta = J_{2c}\Theta_c', \\ X &= J_2X', \quad\quad Y = J_2Y'. \end{aligned} \tag{16}$$

If one uses the notation:

$$M = \begin{bmatrix} M_{1x} & M_{1y} & M_{1\theta} \\ M_{2x} & M_{2y} & M_{2\theta} \\ M_{3x} & M_{3y} & M_{3\theta} \end{bmatrix}, \quad N = \begin{bmatrix} N_{1x} & N_{1y} & N_{1\theta} \\ N_{2x} & N_{2y} & N_{2\theta} \\ N_{3x} & N_{3y} & N_{3\theta} \end{bmatrix}, \tag{17}$$

where M_{1x}, ..., $N_{3\theta}$ are square matrices of $(n + 1)^{st}$ order, then one can obtain expressions for the matrices appearing in (17) from from (5) - (10) of [5], with the help of the presented formulae:

$$\begin{aligned} M_{1x} = [EI_1 \sin^2\varphi + EI_2 \cos^2\varphi] - \omega^2 \{[J_3[m] - J_5(mr)]J_2J_{2j} + \\ + J_1[I_{m2}\cos^2\varphi + I_{m1}\sin^2\varphi]J_{2j}\}, \end{aligned} \tag{18}$$

$$M_{2x} = [(EI_2 - EI_1)\sin\varphi\cos\varphi] - \omega^2 J_1[(I_{m2} - I_{m1})\sin\varphi\cos\varphi]J_{2j}, \tag{19}$$

$$M_{3x} = \omega^2\{J_1[x_p \sin\varphi][s(mr)] + J_1[mx_{\text{т}}\sin\varphi]J_2J_{2j} - J_1[mx_{\text{т}} r\sin\varphi]J_{2j}\}, \tag{20}$$ /34

$$M_{1y} = [(EI_2 - EI_1)\sin\varphi\cos\varphi] - \omega^2 J_1[(I_{m2} - I_{m1})\sin\varphi\cos\varphi]J_{2j}, \tag{21}$$

$$\begin{aligned} M_{2y} = [EI_1\cos^2\varphi + EI_2\sin^2\varphi] + \omega^2\{J_5(mr)J_2J_{2j} - \\ - J_1[I_{m2}\sin^2\varphi + I_{m1}\cos^2\varphi]J_{2j}\}, \end{aligned} \tag{22}$$

$$M_{3y} = -\omega^2\{J_1[x_p\cos\varphi][s(mr)] - J_1[mx_{\text{т}} r\cos\varphi]J_{2j}\}, \tag{23}$$

$$M_{1\theta} = \omega^2\{[s(mr)][x_p\sin\varphi]J_2 - [r]J_1[mx_{\text{т}}\sin\varphi]J_2\}, \tag{24}$$

$$M_{2\theta} = \omega^2\{-[s(mr)][x_p\cos\varphi]J_2 + J_1[mx_{\text{т}} r\cos\varphi]J_2\}, \tag{25}$$

$$\begin{aligned} M_{3\theta} = [GI_c] + \omega^2\{[I_p\alpha_c F][s(mr)] + J_1[mx_{\text{т}} e_0\cos\varphi]J_2 + \\ + J_1[(I_{m2} - I_{m1})\cos 2\varphi]J_2\}, \end{aligned} \tag{26}$$

$$N_{1x} = J_3 [m] J_2 J_{2j} + J_1 [I_{m2} \cos^2 \varphi + I_{m1} \sin^2 \varphi] J_{2j}, \quad (27)$$
$$N_{2x} = J_1 [(I_{m2} - I_{m1}) \sin \varphi \cos \varphi] J_{2j}, \quad (28)$$
$$N_{3x} = -J_1 [m x_{\tau} \sin \varphi] J_2 J_{2j}, \quad (29)$$
$$N_{1y} = J_1 [(I_{m2} - I_{m1}) \sin \varphi \cos \varphi] I_{2j}, \quad (30)$$
$$N_{2y} = J_3 [m] J_2 J_{2j} + J_1 [I_{m2} \sin^2 \varphi + I_{m1} \cos^2 \varphi] J_{2j}, \quad (31)$$
$$N_{3y} = J_1 [m x_{\tau} \cos \varphi] J_2 J_{2j}, \quad (32)$$
$$N_{1\theta} = -J_3 [m x_{\tau} \sin \varphi] J_2, \quad (33)$$
$$N_{2\theta} = J_3 [m x_{\tau} \cos \varphi] J_2, \quad (34)$$
$$N_{3\theta} = J_1 [I_m] J_2. \quad (35)$$

Here, [s (mr)] denotes the diagonal matrix, whose elements are obtained by summing the elements of the corresponding row of the square matrix J_1 [mr].

One must keep in mind that in the case of a rigidly fastened blade, one must use J_2 in place of the matrix J_{2j}; and in the case of a jointed blade with swinging compensation, one must use the matrix J_{2c} in place of J_{2j} in M_{ix}, M_{iy}, N_{ix}, N_{iy} (i = 1, 2, 3), and in place of J_2 in $M_{i\theta}$, $N_{i\theta}$.

Here, Z = {X" Y" Θ'} for the rigid blade, Z_J = {X''_j Y''_j Θ'} for the jointed blade without swinging compensation, and Z_c = {X''_c Y''_c Θ'_c} for the jointed blade with swinging compensation.

For calculation of the blade with swinging compensation, it is necessary to omit the first column and add zeros as the last column in the diagonal matrices $[EI_1 \sin^2 \varphi + EI_2 \cos^2 \varphi]$, $[EI_1 \cos^2 \varphi + EI_2 \sin^2 \varphi]$, $[GI_{c} + \omega^2 [I_p \alpha_c / F] [s(mr)]]$, appearing as terms in the expressions for M_{1x}, M_{2y}, $M_{3\theta}$. One must similarly transform the square matrices $J_1 [x_p \sin \varphi] [s(mr)]$ and $J_1 [x_p \cos \varphi] [s(mr)]$, appearing in M_{3x} and M_{3y}.

When considering the boundary conditions according to which the deflection and twisting torques at the end of the blade are zero, one must omit in the matrices M and N the rows with numbers 1, n + 2, and 2n + 3, and the columns with the same numbers* (in the case of the blade with swinging compensation, only the rows are omitted as the columns dropped out with the use of the special form of the matrix J_{2c}). The three missing equations for /35
the blade with swinging compensation are obtained from the boundary conditions (13). They are taken into account by introducing into the matrix M, three rows of the form:

$$\begin{array}{l|}
\overbrace{0\ldots0}^{n+1}\ \ \overbrace{0\ldots0}^{n}\ k_1\ \overbrace{0\ldots0}^{n}1 \\
\overbrace{0\ldots0}^{n+1}\ \ \overbrace{0\ldots0}^{n-1}10\ \overbrace{0\ldots0}^{n-1}P0 \\
\overbrace{0\ldots0}^{n-1}10\ \overbrace{0\ldots0}^{n+1}\ \ \overbrace{0\ldots0}^{n+1}.
\end{array}$$

Accordingly, three zero rows are introduced in the matrix N in the end.

The blades with other versions of fastening can also be calculated by the technique developed for the blade with swinging compensation. Thus, in the case of the jointed blade without swinging compensation, it is sufficient to set $k_1 = P = 0$; and for calculation of the blade rigidly fastened to the hub, the last three rows of the matrix M must have the form:

$$\begin{array}{l|}
\overbrace{0\ldots0}^{n}1\overbrace{0\ldots0}^{n+1}\ \ \overbrace{0\ldots0}^{n+1} \\
\overbrace{0\ldots0}^{n+1}\ \ \overbrace{0\ldots0}^{n}1\ \overbrace{0\ldots0}^{n+1} \\
\overbrace{0\ldots0}^{n+1}\ \ \overbrace{0\ldots0}^{n+1}\ \ \overbrace{0\ldots0}^{n}1.
\end{array}$$

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*For programming convenience, the corresponding rows and columns can be omitted in the matrix U.

According to the technique of [8], the integrating matrices J are obtained by summing the elements of some matrix L, composed of the weighting factors of the numerical integration formulae.

This matrix is suitably represented in the form:

$$L = \begin{matrix} 0 & 0 & \ldots & 0 \\ L_1 & 0 & \ldots & 0 \\ 0 & L_2 & \ldots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \ldots & L_m \end{matrix} \qquad (36)$$

Here, the first row of zeros, L_i $(i = 1, 2, \ldots, m - 1)$ are rectangular matrices of order $k_i \times (k_i + 1)$, corresponding to the i^{th} interval of the blade with integration spacing h_i, where the number of columns $k_i + 1$* equals the number of calculated cross sections per blade interval (including the cross sections bordering neighboring intervals:

$$L_i = \frac{h_i}{24}\begin{bmatrix} 9 & 19 & -5 & 1 & 0\ldots & 0 & 0 & 0 & 0 & 0 \\ -1 & 13 & 13 & -1 & 0\ldots & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 13 & 13 & -1\ldots & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0\ldots & -1 & 13 & 13 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0\ldots & 0 & -1 & 13 & 13 & -1 \\ 0 & 0 & 0 & 0 & 0\ldots & 0 & 1 & -5 & 19 & 9 \end{bmatrix} \qquad (37)$$

The matrix L is constructed such that the elements of L_i, /36
marked by the diagonal line are located on the main diagonal. In order to take into account the various stagger of vertical,

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*Must have $k_i \geq 3$.

horizontal and axial joints for constructing the matrices M and N, according to Formulae (17) - (35), it is necessary to use different integrating matrices J (we will distinguish them by the indices v, h, and ax) according to the table:

	x		y		θ	
M_1, N_1	$(J_{1,3,5})_v$	$(J_2, J_{2j}, J_{2c})_v$	$(J_{1,3,5})_v$	$(J_2, J_{2j}, J_{2c})_h$	$(J_{1,3,5})_v$	$(J_2, J_{2c})_{ax}$
M_2, N_2	$(J_{1,3,5})_h$	$(J_2, J_{2j}, J_{2c})_v$	$(J_{1,3,5})_h$	$(J_2, J_{2j}, J_{2c})_h$	$(J_{1,3,5})_h$	$(J_2, J_{2c})_{ax}$
M_3, N_3	$(J_{1,3,5})_{ax}$	$(J_2, J_{2j}, J_{2c})_v$	$(J_{1,3,5})_{ax}$	$(J_2, J_{2j}, J_{2c})_h$	$(J_{1,3,5})_{ax}$	$(J_2, J_{2c})_{ax}$

It is advisable to take the same cross sections $r_0, r_1, \ldots, r_{n-1}$ for the matrices with the indices v, h, and ax, and the cross section r_n to coincide, respectively, with the vertical, horizontal, and axial joints. Then the length of the last blade interval equals:

$$h_{nv} = r_{n-1} - r_v; \; h_{nh} = r_{n-1} - r_h; \; h_{nax} = r_{n-1} - r_{ax}, \tag{38}$$

and the matrix L_m has the form for one row:

$$L_{mv} = \left[\frac{h_{nv}}{2} \frac{h_{nv}}{2}\right], \quad L_{mh} = \left[\frac{h_{nh}}{2} \frac{h_{nh}}{2}\right], \; L_{max} = [h_{nax}]. \tag{38'}$$

The cited expression for L_{max} permits introducing into the calculation the elastic torsion mounting at the blade butt, which simulates the elasticity c_b of the blade seal because of the pliability of the skewness automaton. The torsion rigidity of the mounting must be equal:

$$(GI_c)_n = h_{nax}\, c_v. \tag{39}$$

If the natural oscillations of the blade are considered taking into account only the deflection oscillations in the sweeping plane yz, then Equation (2) is replaced by the matrix equation:

$$M_{2y} Y''' - p^2 N_{2y} Y'' = 0. \qquad (40)$$

Blade oscillations with consideration of deflection of the blade in the sweeping plane and of twisting are described by the matrix equation:

$$\begin{bmatrix} M_{2y} & M_{2\theta} \\ M_{3y} & M_{3\theta} \end{bmatrix} \begin{bmatrix} Y'' \\ \Theta' \end{bmatrix} - p^2 \begin{bmatrix} N_{2y} & N_{2\theta} \\ N_{3y} & N_{3\theta} \end{bmatrix} \begin{bmatrix} Y'' \\ \Theta' \end{bmatrix} = 0. \qquad (41)$$

One can similarly write the equations for the natural oscillations of the blade for other combinations of deformations. The matrices M_{2y}, ..., $N_{3\theta}$ introduced into these equations are determined by Formulae (18) - (35), taking into account the cited considerations relating to the effect of the boundary conditions on the form of the integrating matrices and the vectors X", Y", Θ'.

Calculation of the natural oscillations by the discussed technique includes the following steps in the general case:

1) calculation of the matrices M and N from the Formulae (18) - (35), excluding the necessary number of rows and columns;

2) inversion of the matrix M and calculation of U from Formula (5);

3) determination of the eigenvalues and eigenvectors of the matrix U. In the particular cases (40), (41), the number of calculations for 1) is significantly reduced. /37

The discussed technique was programmed for the M-220 computer and multiple calculations were carried out with 18 calculated cross sections. In particular, the calculated results for 18 calculated cross sections were compared with the exact solution for the case of deflection oscillations of a cantilever of constant cross section. The relative error in the frequencies of the natural oscillations for the first four harmonics did not exceed 0.035%, i.e., the accuracy of the technique with 18 calculated cross sections is very high.

A sharp increase in rigidity and linear mass occurs in the butt of an actual blade of a helicopter. To evaluate the error caused by this fact, calculations were carried out for 6 harmonics of blade oscillations with 18 calculated cross sections. In one of the calculations, the blade butt (29.4% of the length) was divided by the calculated cross sections into 9 intervals, and the end — into 8; in another — into 7 and 10 intervals, respectively. The results of these calculations practically coincided; the greatest discrepancy in deflection occurred in the sixth harmonic at the end of the blade and did not exceed 6%. The lower harmonics coincided significantly better, the frequency differences did not exceed 0.5%. Such results of the two calculations with different arrangements of the calculated cross sections permits evaluating the order of calculation errors for actual blades, and reaching the conclusion that blade calculation with 18 calculated cross sections rationally distributed along its length will fully provide the accuracy necessary in practice.

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16. Abstract

Application of the method of integrating matrices to the calculation of the natural vibrations of a propeller blade with allowance for deflection in two planes and for torsion

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